Bode plot

Linear Control

$$T(s) = K\frac{P(s)}{Q(s)} = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$= T_1(s)T_2(s)T_3(s)\cdots\cdots$$

Frequency response:

$$T(j\omega) = T_1(j\omega)T_2(j\omega)T_3(j\omega)\cdots\cdots$$

Let

$$T_i(j\omega) = |T_i(j\omega)|e^{j\theta_i(j\omega)}$$

Then

$$|T(j\omega)| = \prod |T_i(j\omega)|$$

and

$$\theta(j\omega) = \sum \theta_i(j\omega)$$

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$$|G(j\omega)|_{db} \hat{=} 20 log|G(j\omega)|$$

Therefore;

$$|T(j\omega)|_{db} \hat{=} 20 log|T(j\omega)|$$
$$\hat{=} 20 log\left|\prod T_i(j\omega)\right|$$
$$\hat{=} \sum 20 log|T_i(j\omega)|$$
$$= \sum |T_i(j\omega)|_{db}$$

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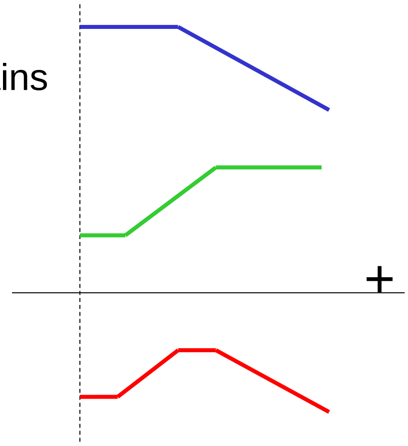
$$T(j\omega) = \prod T_i(j\omega)$$

Overall db gain
=sum of individual db gains

$|T_1|_{db} + |T_2|_{db} = |T|_{db}$



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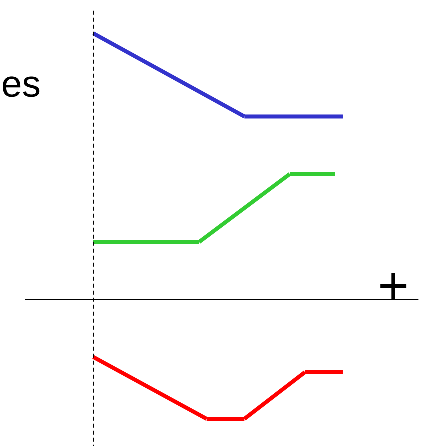
$$T(j\omega) = \prod T_i(j\omega)$$

Overall phase
=sum of individual phases

$\theta_1 + \theta_2 = \theta$



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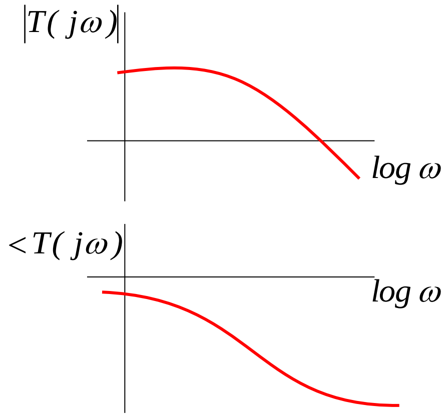
Bode plot

Plot of db magnitude
vs. Log frequency

Plot of phase
vs. Log
frequency



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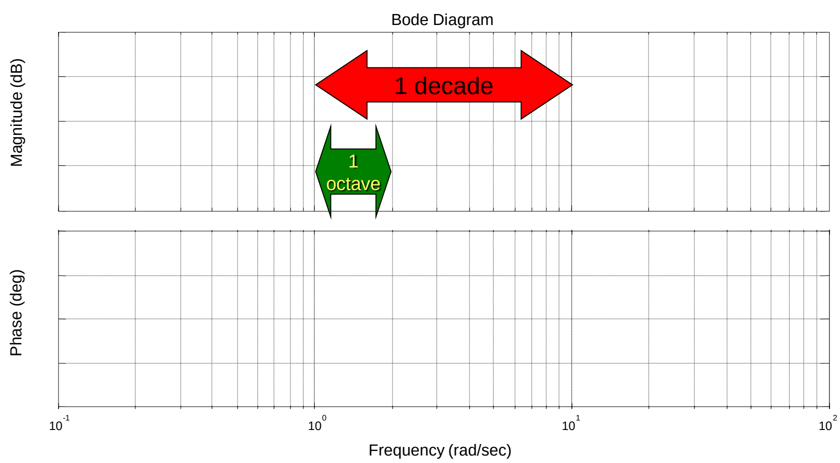
Semi-log paper



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Procedure

- Factorize the transfer function in to "simple" factors
- Obtain magnitude(db) versus frequency and phase versus frequency characteristics for each
- Obtain overall magnitude(db) versus frequency characteristics by adding those of factors
- Obtain overall phase versus frequency characteristics by adding those of factors

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"simple" factors

Constant multiplier	K
Origin zero	s
Finite real zero	$1+ {}^{s}\!/_{z}$
Conjugate zero pair	$1+2\xi\left(\frac{s}{\omega_n}\right)+\left(\frac{s}{\omega_n}\right)^2$
Origin pole	${}^{1}\!/_{s}$
Finite real pole	$\frac{1}{1+ {}^{s}\!/_{p}}$
Conjugate pole pair	$\frac{1}{1+2\xi\left(\frac{s}{\omega_n}\right)+\left(\frac{s}{\omega_n}\right)^2}$

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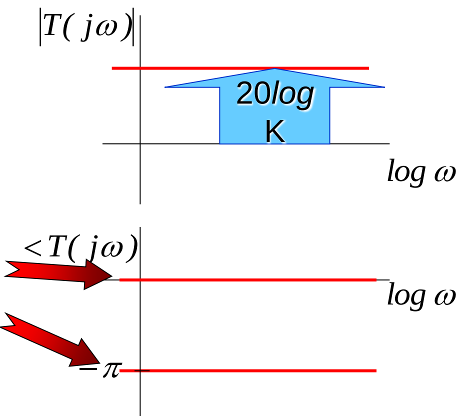
Constant multiplier K

$$|T(j\omega)| = |K|$$

$$|T(j\omega)|_{db} = 20 \log |K|$$

$$< T(j\omega) = < K = \begin{cases} 0 & K > 0 \\ -\pi & K < 0 \end{cases}$$



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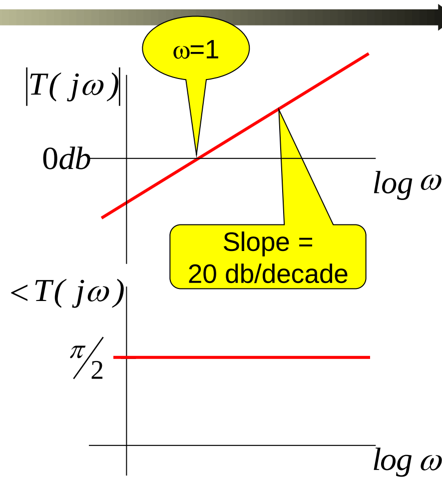
Origin zero: s

$$T(j\omega) = j\omega$$

$$|T(j\omega)| = \omega$$

$$|T(j\omega)|_{db} = 20 \log \omega$$

$$< T(j\omega) = \pi/2$$



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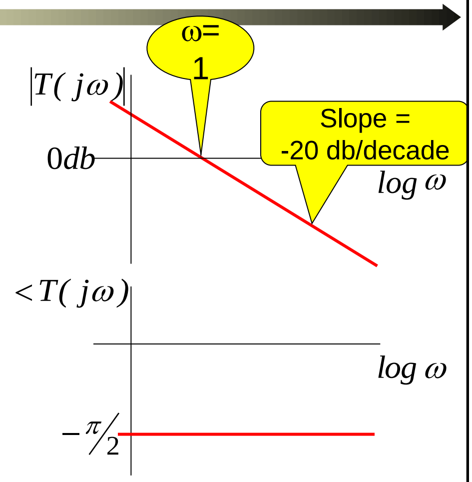
Origin pole: $\frac{1}{s}$

$T(j\omega) = \frac{1}{j\omega}$

$|T(j\omega)| = \frac{1}{\omega}$

$|T(j\omega)|_{db} = -20 \log \omega$

$< T(j\omega) = -\frac{\pi}{2}$



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Finite real zero $T(s) = 1 + \frac{s}{z}$

$T(j\omega) = 1 + \frac{j\omega}{z}$

$|T(j\omega)| = \sqrt{1 + \left(\frac{\omega}{z}\right)^2}$

As $\omega \to 0$ $|T(j\omega)| \to 1$

As $\omega \to \infty$ $|T(j\omega)| \to \left|\frac{\omega}{z}\right|$

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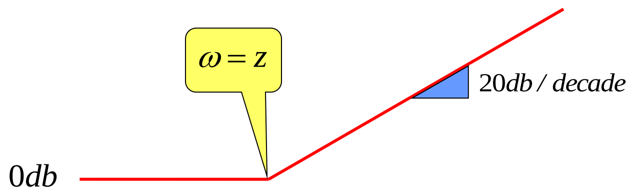
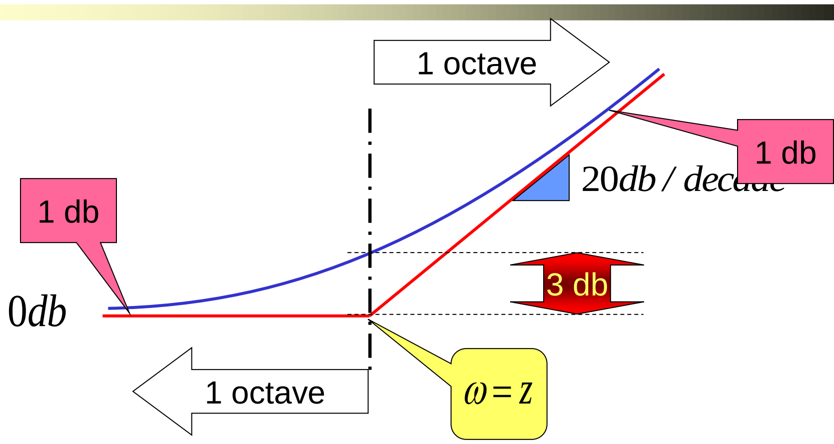
Asymptotes

$$\omega \to 0 \qquad |T(j\omega)| = 1 \qquad |T(j\omega)|_{db} = 0$$

$$\omega \to \infty \qquad |T(j\omega)| \to \left|\frac{\omega}{z}\right| \qquad |T(j\omega)|_{db} \to 20\log\omega - 20\log z$$



Deviations from actual



Finite real zero: Phase vs. Frequency

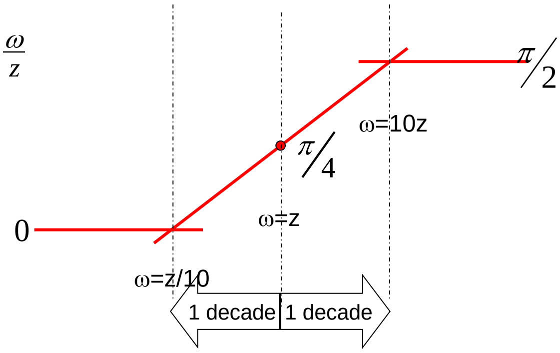
$$T(j\omega)=1+{j\omega}/{z}$$

$$\theta(j\omega)=\angle T(j\omega)=\tan^{-1}\frac{\omega}{z}$$

$$\theta(j\omega)\xrightarrow{\omega\to 0}0$$

$$\theta(j\omega)\xrightarrow{\omega\to\infty}{\pi}/{2}$$

$$\theta(j\omega)\xrightarrow{\omega\to z}{\pi}/{4}$$



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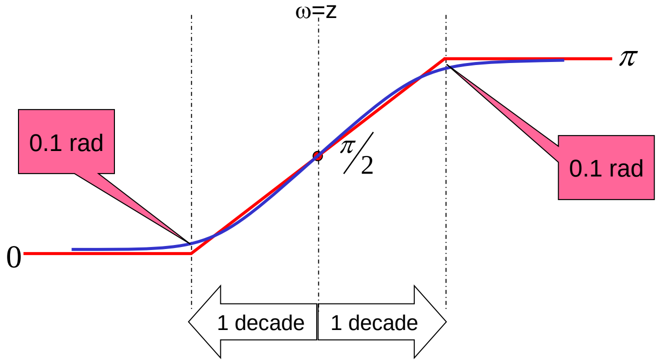

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Finite real pole $T(s) = 1 / \left(1 + s/p\right)$

$$T(j\omega) = 1 / \left(1 + j\omega/p\right) \qquad \left|T(j\omega)\right| = 1 / \sqrt{1 + \left(\frac{\omega}{p}\right)^2}$$

$$\omega \to 0 \qquad \left|T(j\omega)\right| = 1 \qquad \left|T(j\omega)\right|_{db} = 0$$

$$\omega \to \infty \qquad \left|T(j\omega)\right| \to \left|\frac{p}{\omega}\right| \quad \left|T(j\omega)\right|_{db} \to -20\log\omega + 20\log p$$

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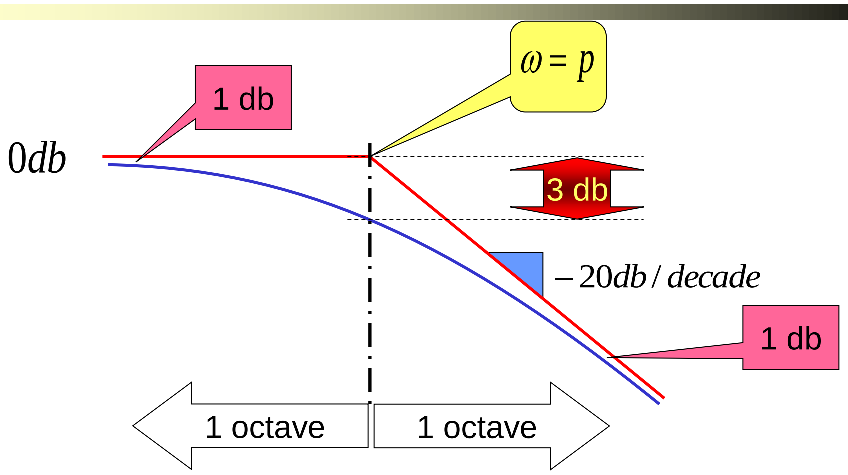

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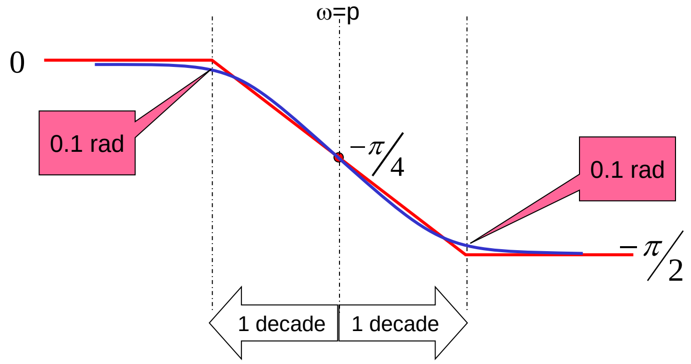
Finite real pole: Phase vs. Frequency

$$T(j\omega) = \frac{1}{1 + j\omega/p} \qquad \theta(j\omega) = \angle T(j\omega) = -\tan^{-1}\frac{\omega}{p}$$



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Conjugate zero pair: $1 + 2\xi\left(\frac{s}{\omega_n}\right) + \left(\frac{s}{\omega_n}\right)^2$

$$T(j\omega) = 1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\left(\frac{\omega}{\omega_n}\right)$$

$$|T(j\omega)| = \sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\left(\xi\frac{\omega}{\omega_n}\right)^2} \quad \rightarrow \begin{cases} 1 & \omega \rightarrow 0 \\ \left(\frac{\omega}{\omega_n}\right)^2 & \omega \rightarrow \infty \end{cases}$$

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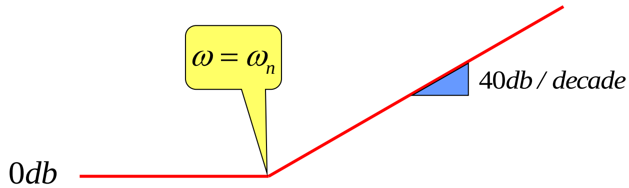
$$|T(j\omega)|_{db} \xrightarrow{\omega \to 0} 20\log 1 = 0$$

$$|T(j\omega)|_{db} \xrightarrow{\omega \to \infty} 20\log\left(\frac{\omega}{\omega_n}\right)^2 = 40\log\omega - 40\log\omega_n$$



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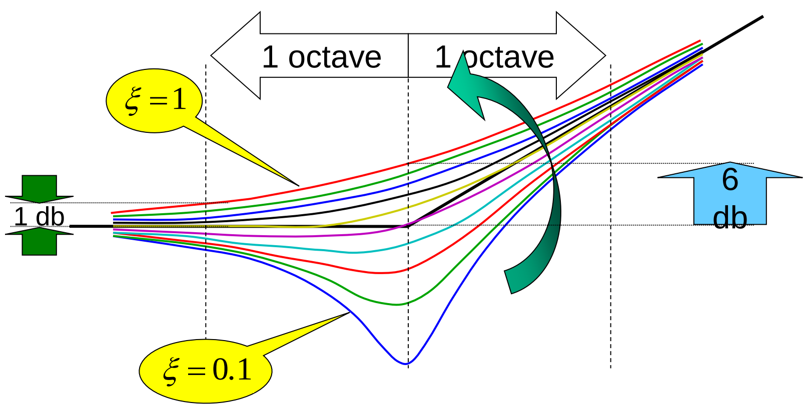
Magnitude vs. frequency



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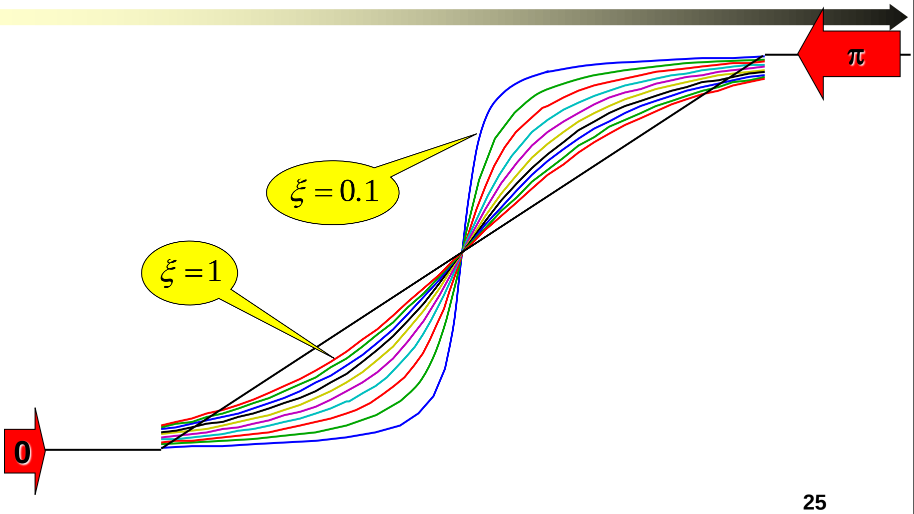
Phase vs. frequency



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Conjugate pole pair: $\frac{1}{1+2\xi\left(\frac{s}{\omega_n}\right)+\left(\frac{s}{\omega_n}\right)^2}$

$$T(j\omega)=\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$|T(j\omega)|=\left(\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\left(\xi\frac{\omega}{\omega_n}\right)^2}\right)^{-1}\rightarrow\begin{cases}1 & \omega\rightarrow 0\\ \left(\frac{\omega_n}{\omega}\right)^2 & \omega\rightarrow\infty\end{cases}$$

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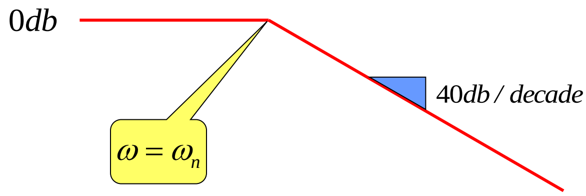
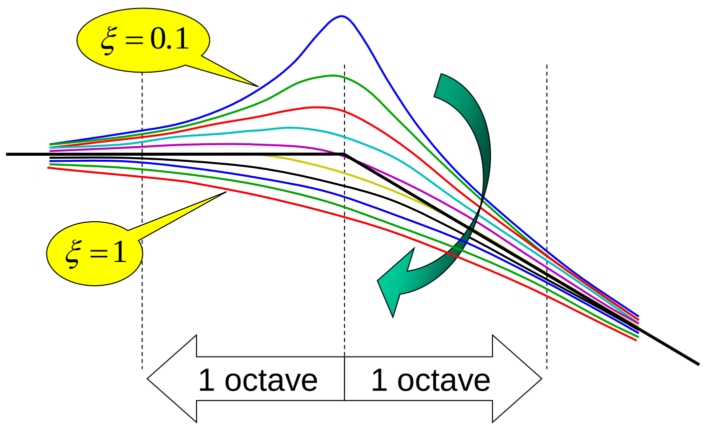
$$|T(j\omega)|_{db} \xrightarrow{\omega\to 0} 20\log 1 = 0$$

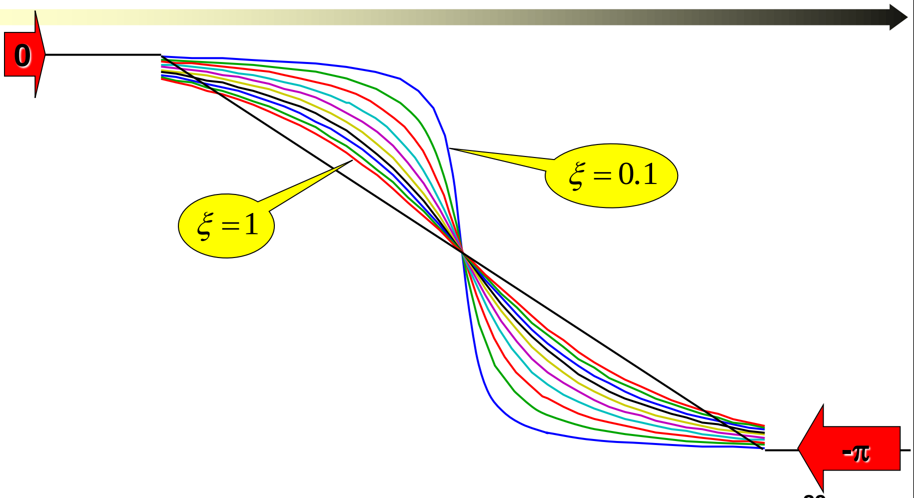
$$|T(j\omega)|_{db} \xrightarrow{\omega\to \infty} 20\log\left(\frac{\omega_n}{\omega}\right)^2 = 40\log\omega_n - 40\log\omega$$



Magnitude vs. frequency



Phase vs. frequency



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Example

$$G(s) = 8\frac{s+1}{s(s^2+2s+4)}$$

$$= 2\frac{s+1}{s\left(1+2\frac{1}{2}\frac{s}{2}+\left(\frac{s}{2}\right)^2\right)}$$

T_1 T_2 T_3 T_4

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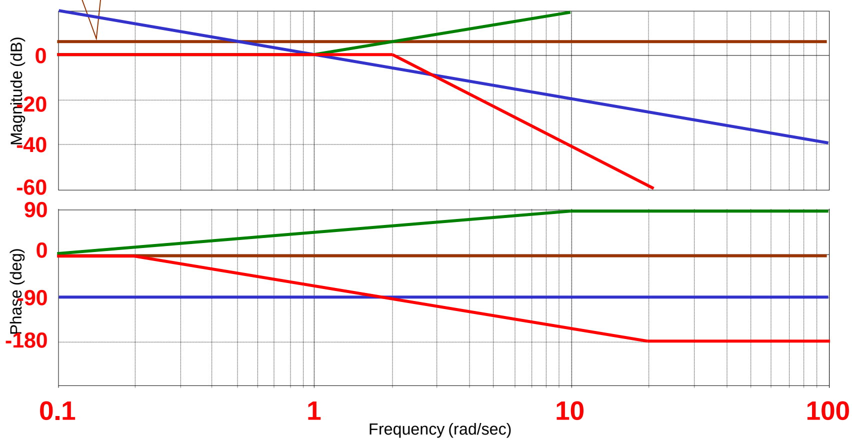
$$G(s) = 2\frac{s+1}{s\left(1+2\frac{1}{2}\frac{s}{2}+\left(\frac{s}{2}\right)^2\right)}$$

$20\log 2 = 6db$



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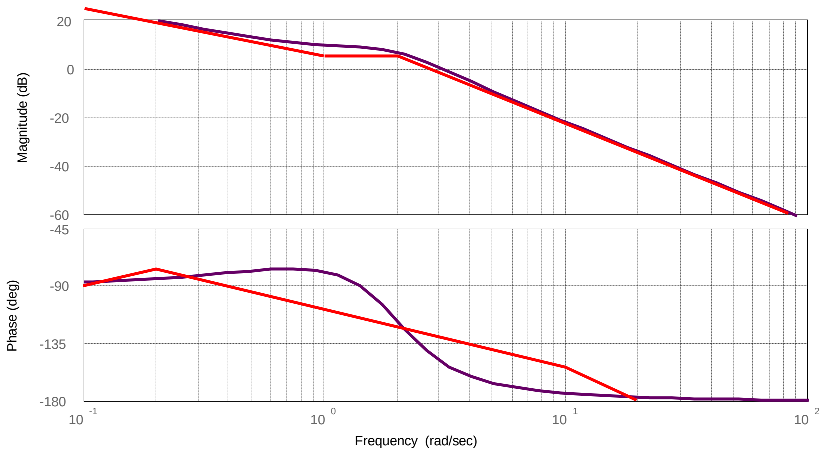

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